Letters to the Editor

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Chiral symmetry breaking and quarks (current and constituent)

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Recently, Gell-Mann (1972) has emphasised that identification of current quarks with constituent quarks lead to many difficulties, so that all the extensive literature, in which the constituent quarks are treated as current quarks, should not be correct. This consideration distinguishes the two groups $U(6)_{W,\ strong}$ and $U(6)_{W,\ current}$, and Gell-Mann (1972) has suggested that they may be related by a unitary transformation which is subsequently investigated by Melosh (1974) in free quark model and is found to work very well (Hey *et al* 1973, Gillman *et al* 1974).

The above mentioned distinction is not made in the Gell-Mann, Oakes and Renner's (1968) (GMOR) determination of the weight parameter c, appearing in the $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$ symmetry breaking Hamiltonian density, viz.,

$$H' = u_0 + cu_8 \qquad \ldots \quad (1)$$

Further, a recent phenomenological analysis of Scadron & Jones (1974), suggests that GMOR solution for c is not consistent with the meson-meson and meson-baryon σ-terms that can be extracted from the experiment. The GMOR solution is also ruled out by the constraints on the $SU(3)\otimes SU(3)$ symmetry breaking parameters derived by Prasad (1974). Thus it is important to make the parametrisation of the $(3,\bar{3})\oplus(\bar{3},3)$ model of chiral symmetry breaking free from the above drawbacks.

The form of eq. (1) was originally suggested by the free quark model (Gell-Mann 1962, 1964) and the parameter c is related to the bare quark masses in the the following way,

$$c = \frac{1}{\sqrt{2}}\,(m_u+m_d-2m_s)/(m_u+m_d+m_s), \qquad \ldots \quad (2)$$

m_u, m_d and m_s being the bare current quark masses. Even though the quarks themselves are presumably fictitious, the above quark masses are important phenomenological parameters indicating how chiral symmetry is broken. Beside c there is another fundamental SU(3)$\otimes$SU(3) symmetry breaking parametter,

$$c' = \xi_8/\xi_0, \quad \text{where} \quad \xi_i = \langle 0 | u_i | 0 \rangle. \qquad \ldots \quad (3)$$

c and c' are generally referred as the *driving ratio* and the *response ratio*, respectively.

To determine these parameters from the pseudoscalar meson masses the following matrix elements of the axial vector divergences are to be conxidered

$$f_i m_i^2 = \langle 0 | \partial_\mu A_i^\mu | P_i \rangle = \langle 0 | [F_i^5, H'] | P_i \rangle$$

$$= (d_{0ii} + c d_{8ii}) Z_i + \sqrt{\frac{2}{3}}\, c \langle 0 | v_0 | P_8 \rangle \delta_{i8}, \qquad \ldots \quad (4)$$

where $Z_i = \langle 0 | v_i | P_i \rangle$, f_i and m_i refer to the decay constant and mass of the pseudoscalar meson P_i. To solve for c it is necessary to know the ratio Z_π/Z_K, for which GMOR considered the three point function

$$F_{ijk}(t) = \langle P_i(p) | u_j | P_k(p') \rangle. \qquad \ldots \quad (5)$$

and took

$$F_{ijk}(t) = \alpha(t) \delta_{j0} \delta_{ik} + \beta(t) d_{ijk} \qquad \ldots \quad (6)$$

$\alpha(0)$ and $\beta(0)$ being determined by Gell-Mann-Okubo mass formula. But, such an assumption is valid only if the u_i's form an octet under SU(3)$_{strong}$, which is not so. However, eq. (6) leads to (Gell-Mann *et al* 1968),

$$f_\pi \approx f_K \approx f_\eta \quad \text{and} \quad Z_\pi \approx Z_K \approx Z_\eta \qquad \ldots \quad (7)$$

and the first relation is experimentally violated. As there exists no Ademollo-Gatto type theorem for the ratio $\zeta = Z_\pi/Z_K$ [i.e., $\zeta = 1 + 0(\epsilon^2)$], there is no fundamental reason for the second relation of eq. (7) to be valid (Scadron *et al* 1974). Considering eq. (4) for pion and Kaon, GMOR solution, $c = -1{\cdot}25$, is obtained from eq. (7). In order to avoid the identification of SU(3)$_{strong}$ and SU(3)$_{current}$, Fuchs (1973) has not used eq. (6), while he has neglected the t-dependence of $F_{ijk}(t)$ in eq. (5) and also used exact pion and Kaon PCAC, like GMOR. Thus, he could equate $F_{ijk}(m_\pi^2)$ with $F_{ijk}(m_K^2)$, finally yielding $f_\pi Z_\pi \approx f_K Z_K$, which in turn cannot reproduce any significantly different result from GMOR. Further, by assuming the validity of both pion and Kaon PCAC, it can be shown (Gell-Mann *et al* 1968, Fuchs 1973) that the vacuum is approximately invariant under SU(3)$_{current}$ rather than SU(3)$_{strong}$ in the works of both GMOR and Fuchs. This means that $c' \approx 0$ in the above schemes. The constraint of Prasad (1974),

$$\frac{(1+a)(a-2)}{a} [a(b^2-4b+1) - 3b(1-b)] \geqslant 0, \qquad \ldots \quad (8)$$

with $a = c/\sqrt{2}$ and $b = c'/\sqrt{2}$, forbids the solutions $c \approx -1\cdot25$ and $c' \approx 0$ and they fall outside the allowed domains for them

In order to overcome these difficulties, let us take soft meson limits for Z_π and Z_K.

$$Z_\pi = \frac{\alpha_\pi}{\sqrt{3}f_\pi}\ (\sqrt{2}\xi_0 + \xi_8), \qquad \ldots (9)$$

$$Z_K = \frac{\alpha_K}{\sqrt{3}f}\ (\sqrt{2}\xi_0 - \tfrac{1}{2}\xi_8), \qquad \ldots (10)$$

where $\alpha_{\pi,K}$ measure the corrections due to the soft pion and kaon limits (Scadron *et al* 1974) Then from eqs (4), (9) and (10), we find,

$$\frac{f_\pi^2 m_\pi^2}{f_K^2 m_K^2} = \frac{\alpha_\pi(\sqrt{2}+c)(\sqrt{2}+c')}{\alpha_K(\sqrt{2}-\frac{1}{2}c)(\sqrt{2}-\frac{1}{2}c')}. \qquad \ldots (11)$$

To solve for c and c' from eq (11), free from the different drawbacks mentioned earlier, let us resort to the suggestive power of free quark model Even though the vacuum expectation values, ξ_i, are individually divergent in the free quark model, c', can be determined by suitable care Following the standard definition of the scalar densities,

$$u_i(x) = \bar{q}(x)\lambda_i q(x),$$

one finds

$$c' = \frac{1}{\sqrt{2}} <0\,|\,\bar{u}u + \bar{d}d - 2\bar{s}s\,|\,0> / <0\,|\,\bar{u}u + \bar{d}d + \bar{s}s\,|\,0>. \qquad \ldots (12)$$

To determine $<0\,|\,\bar{q}_i(x)\, q_i(y)\,|\,0>$ we use Lehman spectral representation, with the restriction that the quarks are non-interacting Thus, only single quark intermediate state can contribute to the vacuum matrix element under consideration

$$<0\,|\,\bar{q}_i(x)q_i(y)\,|\,0> = \frac{1}{(2\pi)^3}\int_0^\infty d^4p \exp\{ip(x-y)\}\theta(p_0)\theta(p^2)m_i\delta(p^2 - m_i^2)$$

or,

$$<0\,|\,\bar{q}_i(x)q_i(x)\,|\,0> = \frac{m_i}{(2\pi)^3}\int_0^\infty \frac{d^3p}{2\sqrt{p^2+m_i^2}}$$

The ratio of the type $<0\,|\,\bar{q}_i(x)q_i(x)\,|\,0>/<0\,|\,\bar{q}_j(x)q_j(x)\,|\,0>$, has been determined by replacing the upper limit of the above integral by R and considering the limiting value of the integral for $R \to \infty$, which turns out to be independent of m. Thus we should have relations like,

$$<0\,|\,\bar{u}u\,|\,0>/<0\,|\,\bar{d}d\,|\,0> = m_u/m_d. \qquad \ldots (13)$$

From eqs. (12) and (13) it follows that c' is related to the bare quark masses in the same way as c in eq. (2), indicating

$$c' = c, \qquad \ldots \quad (14)$$

In the above derivation of eq. (14) we have not identified the current quarks with the constituent quarks. Therefore, in view of eq. (14), we postulate that the *response ratio*, c' is approximately equal to the *driving ratio*, c. Because of the failure of free quark model to exhibit spontaneous breaking of SU(3)⊗SU(3), one should not confuse that the above abstraction commits to a situation where SU(3)⊗SU(3) is not spontaneously broken. In fact, since it is believed that SU(3)⊗SU(3) is spontaneously broken, so that we have massless pseudoscalar mesons and baryons with a common finite mass in the chiral symmetry limit, we are not involved with the actual existence of free quarks and thus our abstraction is not unjustified. An interacting quark model (e.g., quark-gluon model) could give rise to some modification in eq. (14), but phenomenologically we may take eq. (14) to be true to a good approximation and examine its predicting powers.

With $\alpha_\pi/\alpha_K \approx 0{\cdot}6$ (Scadron *et al* 1974), we have from eq. (11),

$$c \approx -0{\cdot}88. \qquad \ldots \quad (15)$$

Z's are no longer independent of i and now,

$$Z_\pi/Z_K \approx \sqrt{\frac{\alpha_\pi}{\alpha_K}} \; \frac{m_\pi}{m_K}. \qquad \ldots \quad (16)$$

The wide experimental support in favour of the new solution for c is apparent from the phenomenological solution for c ($\sim -1{\cdot}0$) by Scadron & Jones (1974). The SU(3)⊗SU(3) generalisation of Goldberger-Treiman relation provides an independent evaluation, $c = -0{\cdot}9 \pm 0{\cdot}1$ (Jones *et al* 1975), in remarkable agreement with eq. (14). c has also been determined from the baryon chiral symmetry parameters by Gunion *et al* (1976) and found to be $c \approx -0{\cdot}8$. Besides all these different supports in favour of the present solution for c, we find that eq. (14) uniquely satisfies the various constraints on c and c' (Okubo & Mathur 1970, Prasad 1974). For $c = c'$ (i.e., $a = b$) the L.H.S of eq. (8) becomes a perfect square, indicating the validity of eq. (14). Further, now it is not necessary to make $f_\pi = f_K$ as in GMOR scheme and the assumption of both pion and Kaon PCAC does not lead to a $SU(3)_{current}$-invariant vacuum. In case of the pion and kaon PCAC correction being same, eq. (11) gives a rather simple solution for c (Sinha & Gautam 1974),

$$c \approx \sqrt{2}\,\frac{f_\pi m_\pi - f_K m_K}{\frac{1}{2} f_\pi m_\pi + f_K m_K} = -0{\cdot}98,$$

which is also reasonably well consistent with the experimental situation.

Note added in proof:

After the completion of this work, we have received a preprint entitled, (*SU*(3) *Symmetry Breaking and PCAC in the Constituent Quark Basis*, from Norman H. Fuchs, in which Fuchs has corrected his previous estimation of c (Fuchs 1973). In his preprint, Fuchs has avoided the use of soft meson limits for pions and kaons by applying Melosh atransformation (Melosh 1974) to determine the SU(3) $_{strong}$ behaviour of the pseudoscalar densities v_i. This yields,

$$\frac{f_K m_K^2}{f_\pi m_\pi^2} = \left(\frac{m_s + \hat{m}}{2\hat{m}}\right) \text{where } \hat{m} = m_u = m_d.$$

so that from eq. (11) we have $\dfrac{\alpha_\pi}{\alpha_K} \approx \dfrac{f_\pi}{f_K}$ and $c \approx -0{\cdot}94$.

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